MATH-SHU 362

Recitation 6: Series Solution of ODE (II)

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Exercise 1. *Classify the following equation at x_0 and x_1 (ordinary, regular singular, irregular singular).*

1. $x^{100}y'' + 2y' + 5y = 0, x_0 = 0, x_1 = 1$;
2. $\cos(x)y'' + e^x y' + 21y = 0, x_0 = 0, x_1 = \frac{\pi}{2}$;
3. $x^2y'' + \sin(x)y' + 6y = 0, x_0 = 0, x_1 = 1$.

Exercise 2. *Determine the radius of convergence (or give a lower bound) for series solution $y = \sum_{n=0}^{\infty} a_n x^n$ for the following equation. Justify precisely your answer.*

1. $y'' + y = 0$;
2. $(4 - x^2)y'' + x^2y' + y = 0$;
3. $(8 + x^3)y'' + \sin(x)y' + (3 + x^2)y = 0$.

Exercise 3. *Determine the general solution of the given differential equation that is valid in any interval not including the singular point.*

1. $x^2y'' + 4xy' + 2y = 0$;
2. $x^2y'' - 3xy' + 4y = 0$;
3. $(x - 2)^2y'' + 5(x - 2)y' + 8y = 0$.

Exercise 4. *Determine the indicial equation for each regular singular point.*

1. $x^2y'' + \frac{1}{2}(x + \sin x)y' + y = 0$;
2. $x^2y'' + 2xy' + 6e^x y = 0$.

Exercise 5. *Let $a_1 = a_2 = 1$ and and $a_{n+2} = a_{n+1} + a_n, n \in \mathbb{N}$. This is the Fibonacci sequence. Let us find an explicit expression for a_n, $n \in \mathbb{N}$. Define $f(x) = \sum_{n=1}^{\infty} a_n x^n$.*

1. *Show that the power series has a positive radius of convergence.*
2. *Show that $f(x) = \frac{x}{1-x-x^2}$.*
3. *Write power series expansion for $f(x)$ and give the explicit value of a_n.*